Peer-to-Peer Power Trading with Voltage and Congestion Management for Distribution Grids

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***Abstract*—In this work, a novel distribution grid level peer-to-peer (P2P) power market framework is proposed. The proposed market framework enables the satisfaction of distribution grid constraints such as the voltage limits and transfer capacity limits, for all P2P power transactions between each producer-consumer pair. An Alternating direction method of multipliers (ADMM)-based coordinating algorithm is proposed in which the P2P power trading is facilitated at the first step, and the second step involves ancillary services market clearing considering the grid constraints and the power injections of the P2P participants, in a distributed and interactive manner. The distribution locational marginal prices are iteratively computed in the process so that the costs associated with P2P energy transactions are allocated to the respective producer-consumer pairs. As such, the proposed algorithm enables optimal P2P power trading while supporting optimal voltage and congestion management of the distribution grid. The case studies illustrate the effectiveness of the proposed ADMM-based coordination algorithm.**

***Index Terms*—ADMM, optimal power flow, peer-to-peer trading, voltage and congestion management.**

I. Introduction

The modern day distribution systems consist of significant amount of distributed energy sources (DERs), termed as producers henceforth. Traditionally, the producers sell (or the consumers purchase) electricity at the feed-in (or retail) tariff specified by the distribution system operator (DSO) [1]. With the increased penetration of DERs and the advancements in the energy storage systems and the communication technologies, the consumers are also willing to trade with the producers directly. This is partially due to the lower marginal costs offered by the DERs and energy storage systems in comparison to the retail prices of the utility as well [2]. The power trading frameworks which enable such transactions are called P2P power markets.

One of the preliminary requirements in P2P power markets is the privacy concerns of its participants [3]. Therefore, fully centralized optimization frameworks are not practical. In conventional P2P power trading approaches, the distribution grid constraints are not considered [3]. In practice, the power flows in distribution grids including the P2P power transactions must respect the power flow equations and the engineering constraints of the distribution grid. However, implementing distributed P2P power markets which satisfy the distribution grid constraints is a challenging task [2], [3].

To address this issue, first-order approximation of power flow constraints are considered in [4] and the energy collectives are used in [5] for P2P power market optimization. However, these methods do not accurately represent the grid constraints and the DSO is not sufficiently incentivized for providing ancillary services and supporting the P2P participants to make grid-feasible power transactions. Reference [6] suggests a method for allocating grid utilization fees (GUFs) for P2P participants. However, such predefined uniform incentives may not be accurately representing the real costs associated with power transactions. Conversely, a practical mechanism for computing GUFs considering the electric distance between the producers and consumers in the P2P power market is proposed in [3]. Here, the electric distance is computed using the decoupled power flow approximation which is not valid for distribution systems due to a higher R/X ratio. Distribution locational marginal price (DLMP) [7] based GUFs have been proposed in [8], which was later extended in [2] with interpretable GUFs via a decomposition of DLMPs. However, both of these methods do not guarantee the convergence of the interactions between the P2P and distribution utility market mechanisms.

To this end, the main contributions of this work are as follows. We propose a *convex* coordinating algorithm based on ADMM to co-optimize the P2P and ancillary markets in an iterative manner with three objectives: 1) Costs of ancillary services associated with the power transactions in the P2P market must be accurately allocated for each producer-consumer pair and those are collected by the DSO. 2) Power injections of the P2P market must satisfy the distribution grid constraints. 3) Privacy concerns of the P2P market participants are preserved. The results illustrate that the proposed ADMM-based (distributed) coordinating algorithm converges to the centralized optimal solution enabling optimal P2P power trading with optimal power flow (OPF) for distribution grids.

II. Market Framework for P2P Trading in Distribution Systems

The proposed P2P trading enabled electricity market framework for distribution grids facilitates the DERs (and consumers) sell (and buy) power to (and from) the DSO or interact in P2P manner depending on the individual preferences. The DSO is considered as an independent agent which is responsible not only to maintain the power balance in the distribution grid while satisfying the grid constraints, but also to support ancillary services (i.e., voltage and congestion management) for P2P transactions at a competitive price. Fig. 1 illustrates the proposed structure of the P2P enabled utility market.

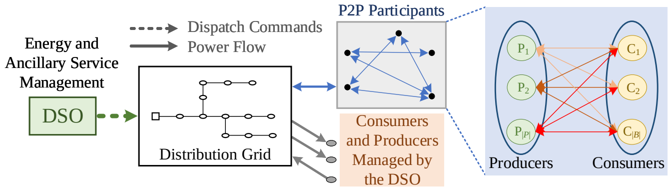

Fig. 1. Distribution system architecture with P2P power trading participants.

A. *Producer and Consumer Model in P2P Power Market*

Let $\mathcal{P}$ be the set of producers and $\mathcal{B}$ be the set of consumers in the P2P power trading market. The objective function of the P2P power market can be formulated as follows

$$\min_{e^{\mathrm{S}},e^{\mathrm{B}},E^{\mathrm{S}},E^{\mathrm{B}}} \mathcal{O}^{\mathrm{P2P}} := \sum_{i\in\mathcal{P}} \mathcal{G}_i\big(e_i^{\mathrm{S}}\big) - \sum_{j\in\mathcal{B}} \mathcal{U}_j\big(e_j^{\mathrm{B}}\big) \tag{1}$$

which is subjected to the following constraints.

$$e^{\mathrm{S}} = E^{\mathrm{S}}\mathbf{1}_{|\mathcal{B}|}, \quad e^{\mathrm{B}} = \big(E^{\mathrm{B}}\big)^{\mathrm{T}}\mathbf{1}_{|\mathcal{P}|} \tag{2a}$$

$$E^{\mathrm{S}} = E^{\mathrm{B}}, \quad E^{\mathrm{S}} \geq 0, \quad E^{\mathrm{B}} \geq 0 \tag{2b}$$

$$\underline{e}^{\mathrm{S}} \leq e^{\mathrm{S}} \leq \bar{e}^{\mathrm{S}}, \quad \underline{e}^{\mathrm{B}} \leq e^{\mathrm{B}} \leq \bar{e}^{\mathrm{B}} \tag{2c}$$

where $\mathcal{G}_i$ is the convex cost function of producer $i \in \mathcal{P}$ and $\mathcal{U}_j$ is the concave welfare function of consumer $j \in \mathcal{B}$. These two functions take the general shapes as explained in [3, Sec. II-A]. The objective function (1) of P2P power trading aims to achieve optimal cost and welfare for producers and consumers respectively; $e^{\mathrm{S}} \in \mathbb{R}_+^{|\mathcal{P}|}$ is the production vector of producers, $e^{\mathrm{B}} \in \mathbb{R}_+^{|\mathcal{B}|}$ is the consumption vector of consumers, and $\big\{E^{\mathrm{S}}, E^{\mathrm{B}}\big\} \in \mathbb{R}_+^{|\mathcal{P}|\times|\mathcal{B}|}$ are the power transaction matrices of producers and consumers respectively. Element (i,j) of E^{S} is the power sold by producer i to consumer j. Element (i,j) of E^{B} is the power purchased by consumer j from producer i. Power balance at each producer and consumer is satisfied by (2a). Constraint (2b) satisfies the compliance of producers' and consumers' power transactions. The production and consumption limits are respected by (2c). Henceforth, $\bar{z}$ and $\underline{z}$ are the maximum and minimum limits of the variable z respectively; z_n is the element n in vector z; $Z_{i,:}$ and $Z_{:,j}$ are the row i and column j of matrix Z respectively; $\mathbf{1}_{(\cdot)}$ is a $(\cdot)\times 1$ dimensional vector of ones; and $(\cdot)^{\mathrm{T}}$ is the transpose of the matrix/vector $(\cdot)$.

B. *Optimal Power Flow in Distribution Grids*

Let $\mathcal{N}$ be the set of buses and $\mathcal{N}_{\mathrm{G}}$ be the set of generators to provide ancillary services in the distribution grid respectively. Let $\mathcal{A}(n)$ and $\mathcal{E}(n)$ be the ancestor bus and the set of descendant buses of bus $n \in \mathcal{N}$ in the (radial) distribution grid respectively. Let $v \in \mathbb{R}^{|\mathcal{N}|}$ be the voltage magnitude vector; $\big\{p^{\mathrm{D}}, q^{\mathrm{D}}\big\} \in \mathbb{R}^{|\mathcal{N}|}$ be the active and reactive power demand vectors; and $\big\{p^{\mathrm{G}}, q^{\mathrm{G}}\big\} \in \mathbb{R}^{|\mathcal{N}_{\mathrm{G}}|}$ be the active and reactive power generation dispatch vectors of the distribution grid respectively. r_n, x_n, P_n, Q_n, and $\bar{S}_n$ be the resistance, reactance, active power flow, reactive power flow, and power transfer capacity of the line between bus $\mathcal{A}(n)$ and bus n respectively. The OPF problem which addresses the operational optimization of the distribution grid is formulated as follows.

$$\min_{p^{\mathrm{G}},q^{\mathrm{G}},v,P,Q} \mathcal{O}^{\mathrm{DSO}} := \sum_{g\in\mathcal{N}_{\mathrm{G}}} \Big[\mathcal{C}_g^{\mathrm{P}}\big(p_g^{\mathrm{G}}\big) + \mathcal{C}_g^{\mathrm{Q}}\big(q_g^{\mathrm{G}}\big)\Big] + \big(v - \tilde{V}\big)^{\mathrm{T}} W \big(v - \tilde{V}\big) \tag{3}$$

where $\mathcal{C}_g^{\mathrm{P}}$ and $\mathcal{C}_g^{\mathrm{Q}}$ are the convex active and reactive power generation cost functions respectively; $\tilde{V}$ is a desired voltage magnitude; and W is a diagonal matrix with weights. The objective function (3) is subjected to following *Linearized DistFlow model*-based constraints [9].

$$\underline{p}^{\mathrm{G}} \leq p^{\mathrm{G}} \leq \bar{p}^{\mathrm{G}}, \quad \underline{q}^{\mathrm{G}} \leq q^{\mathrm{G}} \leq \bar{q}^{\mathrm{G}}, \quad \underline{v} \leq v \leq \bar{v} \tag{4a}$$

$$v_{\mathcal{A}(n)} = v_n + r_n P_n + x_n Q_n; \forall n \in \mathcal{N}\setminus\{1\} \tag{4b}$$

$$P_n^2 + Q_n^2 \leq \bar{S}_n^2; \forall n \in \mathcal{N}\setminus\{1\} \tag{4c}$$

$$p_n = \mathcal{I}_{n,:}^{\mathrm{G}} p^{\mathrm{G}} - p_n^{\mathrm{D}}, \ q_n = \mathcal{I}_{n,:}^{\mathrm{G}} q^{\mathrm{G}} - q_n^{\mathrm{D}}; \forall n \in \mathcal{N} \tag{4d}$$

$$q_n = \sum_{m\in\mathcal{E}(n)} Q_m - Q_n; \forall n \in \mathcal{N} \tag{4e}$$

$$\sum_{i\in\mathcal{P}(n)} e_i^{\mathrm{S}} - \sum_{j\in\mathcal{B}(n)} e_j^{\mathrm{B}} + p_n = \sum_{m\in\mathcal{E}(n)} P_m - P_n; \forall n \in \mathcal{N} \tag{4f}$$

where $\mathcal{I}^{\mathrm{G}} \in \mathbb{R}^{|\mathcal{N}|\times|\mathcal{N}_{\mathrm{G}}|}$ in which element (n,g) of $\mathcal{I}^{\mathrm{G}}$ is 1 if the generator g is connected to bus n or zero otherwise; Variable bounds are enforced in (4a); Equations (4b) satisfy the *Ohm's Law* over the distribution lines; Constraints (4c) respect the power transfer capacity of lines; The active and reactive power balance at buses are satisfied by (4d)-(4f); and $\mathcal{P}(n)$ and $\mathcal{B}(n)$ are the sets of producers and consumers connected at bus n respectively. The co-optimization problem considering the optimal P2P power trading and OPF for the distribution grid is formulated below.

$$\min_{e^{\mathrm{S}},e^{\mathrm{B}},E^{\mathrm{S}},E^{\mathrm{B}}} \mathcal{O}^{\mathrm{P2P}} + \min_{p^{\mathrm{G}},q^{\mathrm{G}},v,P,Q} \mathcal{O}^{\mathrm{DSO}} \tag{5a}$$

$$\text{s.t. } (2),\ (4) \tag{5b}$$

However, the P2P participants are usually reluctant to share certain information like cost/utility functions and the production/consumption capability limitations. Therefore, solving (5) in a centralized manner is not practical [3].

III. ADMM-based Coordinating Algorithm

A. *Decomposition and Decentralized Optimization*

In the following, the co-optimization problem (5) is decomposed based on the ADMM [10]. In this work, the *decentralized* P2P power trading problem and the OPF problem solved by the DSO are considered as the *x-update* and the *z-update* of ADMM respectively. To this end, the optimization problem of producer $i \in \mathcal{P}$ can be formulated as follows.

$$\min_{e_i^{\mathrm{S}},E_{i,:}^{\mathrm{S}}} \mathcal{G}_i\big(e_i^{\mathrm{S}}\big) - \Lambda_i^{\mathrm{S}} e_i^{\mathrm{S}} + \frac{\rho'}{2}\big(e_i^{\mathrm{S}} - \hat{e}_i^{\mathrm{S}}\big)^2 - \Pi_{i,:}^{\mathrm{S}}\big(E_{i,:}^{\mathrm{S}}\big)^{\mathrm{T}} + \frac{\rho}{2}\big(E_{i,:}^{\mathrm{S}} - \hat{E}_{i,:}\big)\big(E_{i,:}^{\mathrm{S}} - \hat{E}_{i,:}\big)^{\mathrm{T}} \tag{6a}$$

$$\text{s.t. } e_i^{\mathrm{S}} - E_{i,:}^{\mathrm{S}}\mathbf{1}_{|\mathcal{B}|} = 0 : \varphi_i^{\mathrm{S}}, \quad E_{i,:}^{\mathrm{S}} \geq 0 : \Omega_{i,:}^{\mathrm{S}} \tag{6b}$$

$$\underline{e}_i^{\mathrm{S}} \leq e_i^{\mathrm{S}} \leq \bar{e}_i^{\mathrm{S}} : \underline{\mu}_i^{\mathrm{S}}, \bar{\mu}_i^{\mathrm{S}} \tag{6c}$$

where $\Pi^{\mathrm{S}} \in \mathbb{R}^{|\mathcal{P}|\times|\mathcal{B}|}$ and $\Lambda^{\mathrm{S}} \in \mathbb{R}_{+}^{|\mathcal{P}|}$ are Lagrangian multipliers introduced in ADMM. ρ and ρ' are the step-sizes used in ADMM. Henceforth, the dual variables are listed after a colon on the right side of the respective constraints with the same dimensions. The optimization problem of consumer $j \in \mathcal{B}$ can be formulated as follows.

$$\min_{e_j^{\mathrm{B}}, E_{:,j}^{\mathrm{B}}} -\mathcal{U}_j\big(e_j^{\mathrm{B}}\big) + \Lambda_j^{\mathrm{B}} e_j^{\mathrm{B}} + \frac{\rho'}{2}\big(e_j^{\mathrm{B}} - \hat{e}_j^{\mathrm{B}}\big)^2$$
$$+ \big(\Pi_{:,j}^{\mathrm{B}}\big)^{\mathrm{T}} E_{:,j}^{\mathrm{B}} + \frac{\rho}{2}\big(E_{:,j}^{\mathrm{B}} - \hat{E}_{:,j}\big)^{\mathrm{T}}\big(E_{:,j}^{\mathrm{B}} - \hat{E}_{:,j}\big) \quad (7a)$$
$$\text{s.t. } e_j^{\mathrm{B}} - \mathbf{1}_{|\mathcal{P}|}^{\mathrm{T}} E_{:,j}^{\mathrm{B}} = 0 : \varphi_j^{\mathrm{B}}, \quad E_{:,j}^{\mathrm{B}} \geq 0 : \Omega_{:,j}^{\mathrm{B}} \quad (7b)$$
$$\underline{e}_j^{\mathrm{B}} \leq e_j^{\mathrm{B}} \leq \bar{e}_j^{\mathrm{B}} : \underline{\mu}_j^{\mathrm{B}}, \bar{\mu}_j^{\mathrm{B}} \quad (7c)$$

where $\Pi^{\mathrm{B}} \in \mathbb{R}^{|\mathcal{P}|\times|\mathcal{B}|}$ and $\Lambda^{\mathrm{B}} \in \mathbb{R}_{+}^{|\mathcal{B}|}$ are Lagrangian multipliers introduced in ADMM. The modified OPF problem solved by the DSO can be formulated as follows.

$$\min_{\substack{p^{\mathrm{G}}, q^{\mathrm{G}}, e^{\mathrm{G}} \\ v, P, Q}} \sum_{g \in \mathcal{N}_{\mathrm{G}}} \Big[\mathcal{C}_g^{\mathrm{P}}(p_g^{\mathrm{G}}) + \mathcal{C}_g^{\mathrm{Q}}(q_g^{\mathrm{G}})\Big] + \big(v - \tilde{V}\big)^{\mathrm{T}} W \big(v - \tilde{V}\big)$$
$$+ \big(\Lambda^{\mathrm{Z}}\big)^{\mathrm{T}} e^{\mathrm{Z}} + \frac{\rho'}{2}\big(e^{\mathrm{Z}} - \hat{e}^{\mathrm{Z}}\big)^{\mathrm{T}}\big(e^{\mathrm{Z}} - \hat{e}^{\mathrm{Z}}\big) \quad (8a)$$
$$\text{s.t. } \sum_{z \in \mathcal{Z}(n)} e_z^{\mathrm{Z}} + p_n = \sum_{m \in \mathcal{E}(n)} P_m - P_n : \varphi_n^{\mathrm{Z}}; \forall n \in \mathcal{N} \quad (8b)$$
$$(4a) - (4e) \quad (8c)$$

where $\mathcal{Z}(n) = \mathcal{P}(n) \cup \mathcal{B}(n)$, $e^{\mathrm{Z}} \in \mathbb{R}^{|\mathcal{P}|+|\mathcal{B}|}$ be the vector of P2P power injections, and $\Lambda_{+}^{\mathrm{Z}} \in \mathbb{R}^{|\mathcal{P}|+|\mathcal{B}|}$ be the vector of Lagrangian multipliers introduced in ADMM. The dual variable φ_n^{Z} is the DLMP of bus n in the distribution system. At the end of each iteration k, producers update the global variables as in (9a) and (9b), consumers update the global variables as in (9a) and (9c), and the DSO updates the global variables as in (9d), where $e^{\mathrm{I}} = \big[\big(e^{\mathrm{S}}\big)^{\mathrm{T}}, \ -\big(e^{\mathrm{B}}\big)^{\mathrm{T}}\big]^{\mathrm{T}}$.

$$\hat{E}_{i,j} = \frac{1}{2}\Big[E_{i,j}^{\mathrm{S}} + E_{i,j}^{\mathrm{B}}\Big]; \forall i \in \mathcal{P}, \forall j \in \mathcal{B} \quad (9a)$$
$$\hat{e}_i^{\mathrm{S}} = \frac{1}{2}\Big[e_i^{\mathrm{S}} + e_i^{\mathrm{Z}}\Big]; \forall i \in \mathcal{P} \quad (9b)$$
$$\hat{e}_j^{\mathrm{B}} = \frac{1}{2}\Big[e_j^{\mathrm{B}} + e_{|\mathcal{P}|+j}^{\mathrm{Z}}\Big]; \forall j \in \mathcal{B} \quad (9c)$$
$$\hat{e}^{\mathrm{Z}} = \frac{1}{2}\Big[e^{\mathrm{Z}} + e^{\mathrm{I}}\Big] \quad (9d)$$

Then, the Lagrange multipliers in corresponding objective functions (6a), (7a), and (8a) are updated as follows.

$$\Pi_{i,:}^{\mathrm{S}}(k+1) = \Pi_{i,:}^{\mathrm{S}}(k) - \rho\big[E_{i,:}^{\mathrm{S}}(k) - \hat{E}_{i,:}(k)\big]; \forall i \in \mathcal{P} \quad (10a)$$
$$\Lambda_i^{\mathrm{S}}(k+1) = \Lambda_i^{\mathrm{S}}(k) + \rho'\big[e_i^{\mathrm{S}}(k) - \hat{e}_i^{\mathrm{S}}(k)\big]; \forall i \in \mathcal{P} \quad (10b)$$
$$\Pi_{:,j}^{\mathrm{B}}(k+1) = \Pi_{:,j}^{\mathrm{B}}(k) - \rho\big[E_{:,j}^{\mathrm{B}}(k) - \hat{E}_{:,j}(k)\big]; \forall j \in \mathcal{B} \quad (10c)$$
$$\Lambda_j^{\mathrm{B}}(k+1) = \Lambda_j^{\mathrm{B}}(k) + \rho'\big[e_j^{\mathrm{B}}(k) - \hat{e}_j^{\mathrm{B}}(k)\big]; \forall j \in \mathcal{B} \quad (10d)$$
$$\Lambda^{\mathrm{Z}}(k+1) = \Lambda^{\mathrm{Z}}(k) + \rho'\big[e^{\mathrm{Z}}(k) - \hat{e}^{\mathrm{Z}}(k)\big] \quad (10e)$$

The proposed ADMM-based coordinating algorithm is summarized in Algorithm 1. The following metrics of residuals are evaluated to examine the *convergence* of the Algorithm 1 [10].

$$\Delta E(k) = E^{\mathrm{S}}(k) - E^{\mathrm{B}}(k), \quad \Delta e(k) = e^{\mathrm{Z}}(k) - e^{\mathrm{I}}(k) \quad (11)$$

When the ADMM-based Algorithm 1 converges satisfying the *termination criteria* in Step 1, the followings hold *true* [10].

$$\Pi^{\mathrm{S}\star} = \Pi^{\mathrm{B}\star}, \ \Lambda_i^{\mathrm{S}\star} = \Lambda_i^{\mathrm{Z}\star}; \forall i \in \mathcal{P}, \ \Lambda_j^{\mathrm{B}\star} = \Lambda_{|\mathcal{P}|+j}^{\mathrm{Z}\star}; \forall j \in \mathcal{B} \quad (12)$$

Algorithm 1: ADMM-based Coordinating Algorithm

Input : $\epsilon, \varepsilon, \Pi^{\mathrm{S}}(0), \Pi^{\mathrm{B}}(0), \Lambda^{\mathrm{S}}(0), \Lambda^{\mathrm{B}}(0), \Lambda^{\mathrm{Z}}(0), \hat{E}(0), \hat{e}^{\mathrm{S}}(0), \hat{e}^{\mathrm{B}}(0), \hat{e}^{\mathrm{Z}}(0), k = 0$

1 **while** $\big\|\Delta E(k)\big\|_\infty \geq \epsilon$ & $\big\|\Delta e(k)\big\|_\infty \geq \varepsilon$ **do**
2 Producers and consumers individually execute (6) and (7) respectively: *x-update*.
3 DSO executes (8): *z-update*.
4 Producer i will send $E_{i,j}^{\mathrm{S}}$ to consumer j and e_i^{S} to DSO; Consumer j will send $E_{i,j}^{\mathrm{B}}$ to producer i and e_j^{B} to DSO; DSO will send e_z^{G} to P2P agents.
5 Global variables will be updated as in (9)
6 Lagrangian multipliers will be updated as in (10).
7 $k \leftarrow k + 1$
8 **end**

Output: $\Pi^{\mathrm{S}\star}, \Pi^{\mathrm{B}\star}, \Lambda^{\mathrm{S}\star}, \Lambda^{\mathrm{B}\star}, \Lambda^{\mathrm{G}\star}, E^{\mathrm{S}\star}, E^{\mathrm{B}\star}, e^{\mathrm{S}\star}, e^{\mathrm{B}\star}, e^{\mathrm{G}\star}$

B. Market Clearing and Revenue Allocation

The *optimal solution* of (6) for all $i \in \mathcal{P}$ will *always* satisfy the following two Karush–Kuhn–Tucker (KKT) conditions.

$$\frac{\partial \mathcal{G}_i\big(e_i^{\mathrm{S}\star}\big)}{\partial e_i^{\mathrm{S}}} - \Lambda_i^{\mathrm{S}\star} + \rho'\big(e_i^{\mathrm{S}\star} - \hat{e}_i^{\mathrm{S}}\big) + \varphi_i^{\mathrm{S}} + \bar{\mu}_i^{\mathrm{S}} - \underline{\mu}_i^{\mathrm{S}} = 0 \quad (13a)$$
$$-\Pi_{i,j}^{\mathrm{S}\star} + \rho\big(E_{i,j}^{\mathrm{S}\star} - \hat{E}_{i,j}\big) - \varphi_i^{\mathrm{S}} - \Omega_{i,j}^{\mathrm{S}} = 0; \forall j \in \mathcal{B} \quad (13b)$$

As the optimal solution satisfies the termination criteria, solving (13) while assuming $e_i^{\mathrm{S}\star}$ and $E_{i,:}^{\mathrm{S}\star}$ are non-binding deduces

$$\Pi_{i,j}^{\mathrm{S}\star} = \frac{\partial \mathcal{G}_i\big(e_i^{\mathrm{S}\star}\big)}{\partial e_i^{\mathrm{S}}} - \Lambda_i^{\mathrm{S}\star}; \forall j \in \mathcal{B}. \quad (14)$$

Similarly, considering the KKT conditions of (7) for all $j \in \mathcal{B}$, and assuming that $e_j^{\mathrm{B}\star}$ and $E_{:,j}^{\mathrm{B}\star}$ are non-binding results in

$$\Pi_{i,j}^{\mathrm{B}\star} = \frac{\partial \mathcal{U}_j\big(e_j^{\mathrm{B}\star}\big)}{\partial e_j^{\mathrm{B}}} - \Lambda_j^{\mathrm{B}\star}; \forall i \in \mathcal{P} \quad (15)$$

When the ADMM-based approach converges, $\Pi_{i,j}^{\mathrm{S}\star} = \Pi_{i,j}^{\mathrm{B}\star}$ for all $i \in \mathcal{P}$ and $j \in \mathcal{B}$. That deduces the following

$$\frac{\partial \mathcal{U}_j\big(e_j^{\mathrm{B}\star}\big)}{\partial e_j^{\mathrm{B}}} = \frac{\partial \mathcal{G}_i\big(e_i^{\mathrm{S}\star}\big)}{\partial e_i^{\mathrm{S}}} + \Big[\Lambda_j^{\mathrm{B}\star} - \Lambda_i^{\mathrm{S}\star}\Big]; \forall i \in \mathcal{P}, \forall j \in \mathcal{B} \quad (16)$$

Let producer i and consumer j be connected to buses n and m of the distribution grid respectively. When the algorithm terminates, $\varphi_m^{\mathrm{Z}} = \Lambda_j^{\mathrm{B}}$ and $\varphi_n^{\mathrm{Z}} = \Lambda_i^{\mathrm{S}}$ which can be deduced from the KKT conditions of (8) (similar to the derivations in (13)-(14)). Hence, when $e_i^{\mathrm{S}\star}$ and $e_j^{\mathrm{B}\star}$ are within their capability limits, the P2P transaction is facilitated *only* when the marginal welfare of consumer j is *sufficient* to satisfy the marginal cost of producer i and the p. u. cost of ancillary services required to enable the power transaction, i.e., $\Lambda_j^{\mathrm{B}} - \Lambda_i^{\mathrm{S}} = \varphi_m^{\mathrm{Z}} - \varphi_n^{\mathrm{Z}}$. Hence, the cost of ancillary services are still covered in the proposed approach although the DLMPs are not shared among the participants in the proposed P2P power market. For all $i \in \mathcal{P}$ and $j \in \mathcal{B}$, consumer j will pay $\Lambda_j^{\mathrm{B}\star} e_j^{\mathrm{B}\star}$ to the DSO, producer i will receive $\Lambda_i^{\mathrm{S}\star} e_i^{\mathrm{S}\star}$ from the DSO, and consumer j will pay $\Pi_{i,j}^{\mathrm{B}\star} E_{i,j}^{\mathrm{B}\star}$ (typically < 0) to producer i. As per (14) and (15), $\Pi_{i,j}^{\mathrm{S}\star}$ and $\Pi_{i,j}^{\mathrm{B}\star}$ would be *negative values* since the DLMPs φ_m^{Z} and φ_n^{Z} (and hence the Λ_j^{B} and Λ_i^{S}) are typically greater than the marginal welfare $\frac{\partial \mathcal{U}_j(e_j^{\mathrm{B}\star})}{\partial e_j^{\mathrm{B}}}$ of consumer j and the marginal cost $\frac{\partial \mathcal{G}_i(e_i^{\mathrm{S}\star})}{\partial e_i^{\mathrm{S}}}$ of producer i respectively.

Therefore, the involvement of DSO in the proposed ADMM-based coordinating algorithm is only by iteratively sharing the P2P power injections e^{Z} with each P2P participant. In each iteration, these injections are computed in (8) while satisfying the distribution grid constraints. As such, the infeasible P2P power injections will be avoided. Further, as shown above, the proposed P2P power market framework clears at the optimal solution which allocates the cost of ancillary services required to enable the P2P power transactions to the respective producer-consumer pairs accurately.

IV. Results and Discussion

The proposed ADMM-based coordinating algorithm is tested on an IEEE 33-bus system which has a total static load of $3.715\,\mathrm{MW}$ and $2.3\,\mathrm{Mvar}$ [11]. The active and reactive static loads are doubled and the transfer capacity of lines are set to $3.5\,\mathrm{MVA}$ to illustrate the effectiveness of the proposed ADMM-based coordinating algorithm. The participants of the P2P market and ancillary services market is documented in Tables I and II. Voltage bounds are set to $[0.97, 1.03]\,\mathrm{p.u.}$; weights of W are set to $10\,\$/\mathrm{h}$; and $\tilde{V} = 1.0$. The ADMM parameters $\rho = 0.5$ for (6) and (7); $\rho' = 12$ for (8); $\epsilon = 10^{-5}$; and $\varepsilon = 10^{-4}$. Algorithm 1 and the individual optimization problems (6), (7), and (8) were programmed in MATLAB and solved using GUROBI. All the simulations were performed on a desktop PC with an Intel® Core i7-4770U four-core CPU processor running at $3.40\,\mathrm{GHz}$ with $8\,\mathrm{GB}$ of RAM.

TABLE I
P2P Market Participants

Producers		P1	P2	P3	P4	P5	P6	
Bus Index		18	22	25	31	2	6	
Capacity (MW)		$[0, 1.6]$	$[0, 2.3]$	$[0, 2.9]$	$[0, 2.5]$	$[0, 2.5]$	$[0, 3.5]$	
Cost	($/MW^{2}h)	0.03	0.02	0.03	0.04	0.05	0.06	
Coeff.	($/MWh)	3.2	4.0	3.0	4.5	3.2	3.8	
Consumers		C1	C2	C3	C4	C5	C6	C7
Bus Index		4	7	11	15	20	24	30
Capacity (MW)		$[0, 2.2]$	$[0, 1.3]$	$[0, 1.6]$	$[0, 1.7]$	$[0, 1.5]$	$[0, 2.5]$	$[0, 2.4]$
Cost	($/MW^{2}h)	-0.10	-0.20	-0.05	-0.05	-0.05	-0.10	-0.10
Coeff.	($/MWh)	4.5	5.0	5.0	4.8	4.0	5.0	5.0

TABLE II
Ancillary Services Market Participants

Bus Index		1	17	21	33	10	24
P/Q-Capacity (MW/Mvar)		$[0, \infty]$	$[0, 3.5]$	$[0, 3.5]$	$[0, 3.9]$	$[-0.3, 0]$	$[-0.6, 0]$
P-Cost	($/MW^{2}h)	0.0	0.2	0.1	0.2	0.3	0.2
Coeff.	($/MWh)	17	20	21	22	-21	-20
Q-Cost	($/Mvar^{2}h)	0.0	0.04	0.02	0.02	0.06	0.04
Coeff.	($/Mvarh)	3	4	5	5	-4	-4

Fig. 2 shows the convergence trajectories of the infinity norms of the residuals of each consumer, producer, and the DSO. Algorithm 1 takes 958 iterations to converge. It was observed that the average computation times of (6), (7), and (8) per iteration were $1.86\,\mathrm{ms}$, $1.88\,\mathrm{ms}$, and $14.13\,\mathrm{ms}$ respectively.

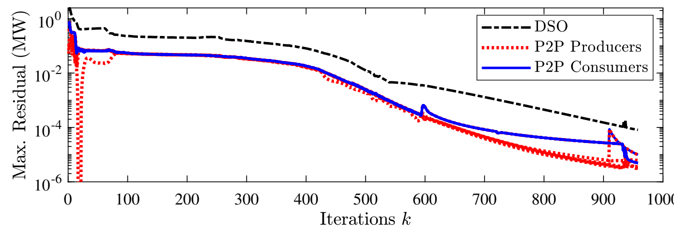

Fig. 2. Convergence trajectories of the residuals $\|\Delta e(k)\|_\infty$ of DSO, and $\|\Delta E(k)\|_\infty$ of P2P producers and P2P consumers over the iterations.

A. Satisfaction of Distribution Grid Constraints

To illustrate the impact of P2P power injections on the voltage and congestion management the simulations were conducted under two scenarios. In Scenario 1, (1) was solved subject to (2). In Scenario 2, (5) was solved in a distributed manner following the procedure in Algorithm 1. It can be observed in Fig. 3 that the power consumption and production of the P2P participants, for instance, the power consumption of C1 and C5, and the power production of P2 and P4, are *significantly different* in the two scenarios. It was further observed that the distribution grid constraints mentioned in (4) were *not satisfied* for the P2P power injections computed under Scenario 1, although those were *satisfied* for the P2P power injections computed under Scenario 2. Hence, it can be concluded that the proposed ADMM-based coordinating algorithm drives the P2P power trading computations towards the distribution grid constraint satisfaction. Further, P2P power transactions computed without considering their impact on distribution grid constraints can be *infeasible* in practice. Moreover, it was observed that the proposed ADMM-based

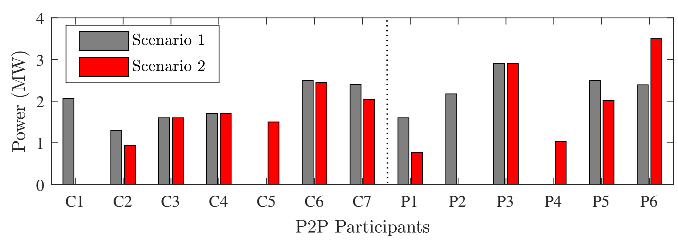

Fig. 3. Consumption and production of consumers (C1-C7) and producers (P1-P6) respectively for Scenario 1 and 2.

coordinating algorithm converges to the optimal solution of the co-optimization problem (5). Table III reports the power transactions between the participants in the P2P market under Scenario 2.

TABLE III
P2P Power Transactions $(E^{\mathrm{S}} = E^{\mathrm{B}})$

(MW)	C1	C2	C3	C4	C5	C6	C7
P1	0	0.0006	0.0611	0.0942	0.0193	0.3479	0.2493
P2	0	0	0	0	0	0	0
P3	0	0.2323	0.4422	0.4754	0.3904	0.7291	0.6305
P4	0	0.0846	0.1951	0.1951	0.2310	0.1842	0.1405
P5	0	0.1138	0.2873	0.3205	0.2436	0.5742	0.4756
P6	0	0.5023	0.6143	0.6148	0.6156	0.6098	0.5431

TABLE IV
GRID UTILIZATION FEES: $\mathbf{1}_{|\mathcal{P}|}\left(\Lambda^{\mathrm{B}}\right)^{\mathrm{T}} - \Lambda^{\mathrm{S}}\mathbf{1}^{\mathrm{T}}_{|\mathcal{B}|}$

(\$/h)	C1	C2	C3	C4	C5	C6	C7
P1	1.254	1.380	1.585	0.770	0.149	1.265	1.346
P2	1.101	1.227	1.432	0.617	−0.005	1.111	1.192
P3	−0.009	0.117	0.321	−0.493	−1.115	0.001	0.082
P4	−0.082	0.044	0.249	−0.566	−1.187	−0.071	0.010
P5	1.099	1.225	1.430	0.615	−0.006	1.109	1.191
P6	−0.112	0.013	0.218	−0.597	−1.218	−0.102	−0.021

B. Market Equilibrium Analysis

GUFs associated between producer-consumer pairs are reported in Table IV. Therein, the negative values imply that the respective P2P power transaction improves the voltage profile or reduces the network congestion of the distribution system and vice versa. Fig. 4 shows the trading price decomposition between consumer 6 and the set of producers. Consumers in P2P power trading only prefer to trade when the marginal costs of the offers (i.e., the $\star$ markers in Fig. 4) are less than their marginal welfare. It can be observed that the marginal generation cost of producer 2 is lower than that of producer 4. However, due to the negative GUFs between consumer 6 and producer 4 compared to the significant GUFs between consumer 6 and producer 2, enforces consumer 6 to purchase power from producer 4 and abandon producer 2. As such, the proposed method promotes P2P participants to form trading pairs which supports voltage and congestion management of the distribution grid.

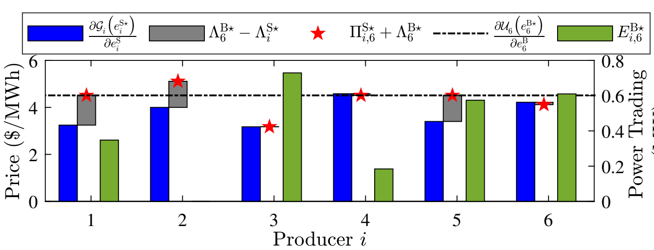

Fig. 4. Marginal cost decomposition and power trading between producers and consumer 6.

Fig. 5 shows the trading price decomposition between producer 5 and the set of consumers. Although the marginal welfare offered by consumer 1 is higher than that of consumer 5, due to significantly higher GUFs, producer 1 avoids making power transactions with consumer 1. Consumers 4 and 5 offer higher marginal welfare have reached their maximum consumption limits which restrict further power transactions between producer 5 and each of them.

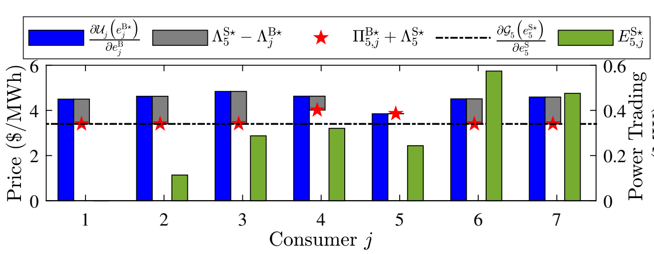

Fig. 5. Marginal cost decomposition and power trading between consumers and producer 5.

V. CONCLUSIONS

In this work, a novel ADMM-based coordinating algorithm is proposed to enable distributed P2P power transactions in distribution grid level. Therein, each producer/consumer participant in the P2P power market and the distribution system operator solve individual optimization problems in an interactive manner while respecting the privacy concerns of each participant. The case studies were conducted on the modified IEEE 33-bus system. The results illustrate that the proposed ADMM-based coordinating algorithm converge to the optimal solution with sufficient accuracy in an acceptable computation time. It was observed that the P2P power transactions computed without considering the distribution grid constraints are infeasible. Further, the proposed algorithm enabled the P2P participants to iteratively evaluate the cost of ancillary services required to enable each P2P power transaction in the distributed computational process (market mechanism) which determines the equilibrium.

ACKNOWLEDGMENT

This work was supported by the Singapore Ministry of Education Academic Research Fund Tier 1 Grant (MOE2018-T1-002-093) and the Agency for Science, Technology and Research (A*STAR) through its Singapore-Germany Academic-Industry (2+2) International Collaboration under Grant A1990b0060.

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